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A note on the Seiberg-Witten solution of N=2 Super Yang-Mills Theory

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ABSTRACT

We examine the Seiberg-Witten treatment of $N = 2$ super Yang-Mills theory, and note that in the strong coupling region of moduli space, some massive particle excitations appear to have negative norm. We discuss the significance of our observation.

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Recently, Seiberg and Witten solved the low energy limit of $N = 2$ super Yang-Mills theory with gauge group $SU(2)$ [1]. Their solution is described by a single holomorphic function $\mathcal{F}$, which determines the geometry of the Kähler-vector multiplet [2], and is generated by one-loop and instanton effects. The vacuum of $N = 2$ super Yang-Mills theory is described by a complex parameter u that describes the spontaneous symmetry breaking from $SU(2)$ to $U(1)$. They explored in great detail the couplings of the $U(1)$ gauge multiplet to the low-lying excitations of the theory and uncovered a rich structure involving duality. They also briefly mentioned the couplings to some of the massive states. In this note we push some of those ideas a little further, and find a surprising result.

Seiberg and Witten compute $\mathcal{F}$ as a function of the chiral $N = 2$ superfield $\mathcal{A}$ describing the unbroken $U(1)$ gauge group. This function determines the low energy physical parameters of the theory. For example,

$$\tau \equiv \langle \mathcal{F}'' \rangle = \frac{\theta}{2\pi} + i\frac{4\pi}{g^2} \tag{1}$$

gives the gauge coupling g^2 in terms of $Im(\langle \mathcal{F}'' \rangle)$. This is positive because τ can be interpreted as the modular parameter of a family of tori, and takes its values in the upper half-plane.

Seiberg and Witten also note that new phenomena occur when

$$Im(\left\langle \frac{\mathcal{F}'}{\mathcal{A}} \right\rangle) = 0 \ . \tag{2}$$

They suggest, and it has been verified numerically, that this occurs on a curve (topologically a circle) in the space of vacua [3]. Outside of that curve, the semi-classically derived spectrum holds, and $Im(\left\langle \frac{\mathcal{F}'}{\mathcal{A}} \right\rangle) > 0$; inside, states may disappear and/or appear, and $Im(\left\langle \frac{\mathcal{F}'}{\mathcal{A}} \right\rangle) < 0$.

As they note, $SU(2)$ gauge invariance implies that $\mathcal{F}(\mathcal{A}) \equiv \mathcal{F}(\sqrt{\mathcal{A}^i \mathcal{A}^i})$, where $\mathcal{A}^i$ is the triplet of gauge superfields of the full group $SU(2)$. Then instead of a complex constant τ, one finds a 3×3 matrix of couplings

$$\tau_{ij} \equiv \left\langle \frac{\partial^2}{\partial \mathcal{A}^i \partial \mathcal{A}^j} \mathcal{F}(\sqrt{\mathcal{A}^k \mathcal{A}^k}) \right\rangle = \left\langle \mathcal{F}'' \frac{\mathcal{A}^i \mathcal{A}^j}{\mathcal{A}^k \mathcal{A}^k} + \frac{\mathcal{F}'}{\sqrt{\mathcal{A}^k \mathcal{A}^k}} (\delta^{ij} - \frac{\mathcal{A}^i \mathcal{A}^j}{\mathcal{A}^l \mathcal{A}^l}) \right\rangle \ . \tag{3}$$

What was apparently overlooked in [1] is the nice decomposition (3) of τ_{ij} into color projection operators onto the $U(1)$ and the $SU(2)/U(1)$ generators. The coefficient of the $U(1)$ fields is just $\langle \mathcal{F}'' \rangle$ (*i.e.*, τ), which, as noted in [1], leads to a positive kinetic energy. The coefficient of the coset fields is instead $\left\langle \frac{\mathcal{F}'}{\mathcal{A}} \right\rangle$ (in the notation of [1], this is also a_D/a). As noted above, the imaginary part of this vanishes on a curve in the space of vacua; on that curve, we expect the corresponding gauge fields to become nonpropagating, auxiliary degrees of freedom [4]. Inside the curve, the would-be particles have negative kinetic terms. This signals a breakdown of the formalism for the coupling to the charged massive W vector gauge multiplets. Presumably, it is a signal that the W particles have disappeared from the spectrum (as anticipated in [1]); however, from the effective action, we do not know how to prove that negative norm states are safely nonexistent rather than a signal of a genuine breakdown of the theory.

Duality transformations cannot illuminate the difficulty, as they map the inside of the curve into itself, and thus do not mix the semiclassical region with this strongly coupled region.

We end with a few comments: Our results do not in any way challenge the validity of the solution of [1] for the low energy excitations; it seems to be an added bonus that in the semiclassical region, the solution also describes the coupling to the massive W's correctly, but there is no reason for this to hold in the strong coupling domain, as the masses are above the cutoff in the Wilson effective action. The question that we have stumbled on in the context of $N = 2$ super Yang-Mills theory is quite general: when do pathologies of the effective action reflect a breakdown of the theory, and when do they represent a signal that the states described by the effective action have disappeared?

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References

[1] N. Seiberg and E. Witten, Nucl. Phys. **B426** (1994) 19.

[2] S. J. Gates, Jr., Nucl. Phys. **238** (1984) 349;
B. de Wit, P. G. Lauwers, R. Philippe, S.-Q. Su and A. Van Proeyen, Phys. Lett. **134B** (1984) 37;
M. Günaydin, G. Sierra and P. K. Townsend, Nucl. Phys. **B242** (1984) 244.

[3] J. de Boer, D. Jatkar, C. Skenderis, B. Peeters, unpublished.
Argyres and Faraggi, unpublished (E. Witten, private communication).

[4] J. Grundberg, U. Lindström, and M. Roček, in preparation.
See also B. Julia and J. F. Luciani, Phys. Lett. **90B** (1980) 270.
C. M. Hull, A. Karlhede, U. Lindström and M. Roček, Nucl. Phys. **B266** (1986) 1.